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Simple Guidance Scheme for Low-Thrust Orbit Transfers

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Introduction

SPACE missions using solar electric propulsion (SEP) will, it is hoped, become more commonplace after SEP technology is successfully demonstrated by the first new millennium mission, Deep Space 1. Much of the previous low-thrust flight mechanics research has been devoted to computing optimal (minimum-fuel or minimum-time) trajectories. Examples include transfers from low Earth orbit (LEO) to geosynchronous orbit (GEO).[1–3] By comparison, the volume of work on guidance laws for electric propulsion spacecraft is somewhat limited. Early examples of low-thrust guidance schemes for lunar and interplanetary missions are presented by Battin[4] and Breakwell and Rauch,[5] respectively. More recent low-thrust guidance methods are demonstrated in Refs. 6–8.

Trajectory optimization methods often utilize a calculus of variations approach, which obtains the optimal trajectory by solving a corresponding two-point boundary-value problem (TPBVP). As a result of the TPBVP solution, the Euler–Lagrange or costate equations define the thrust vector steering control during the optimal transfer in an open-loop fashion. For realistic onboard guidance schemes, the implementation of the optimal control defined by the costate equations may not be feasible or practical for the very long duration transfers performed by low-thrust spacecraft. For example, a typical LEO–GEO transfer using SEP would require hundreds of days of continuous thrusting and thousands of revolutions. This presents several challenges for designing a guidance system for steering the thrust vector. In addition, the level of ground support is significantly increased if frequent uplinks of new guidance commands for thrust steering is necessary. Therefore, a simple autonomous guidance scheme (which can be easily stored and implemented on board the spacecraft) that provides near-optimal performance would be highly beneficial for low-thrust mission operations.

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This Note presents a simple guidance scheme for performing low-thrust orbital transfers. The guidance is based on optimal control laws that are developed by investigating the nature of the variational equations for the orbital elements. The individual control laws are blended so that a simultaneous change of several orbit elements is enacted. A numerical simulation of a low-thrust LEO–GEO transfer is presented to demonstrate the proposed guidance scheme.

Guidance Scheme

Control Laws

The proposed guidance method is based on a combination of individual optimal control laws that maximize the time rate of change of a desired orbital element. To demonstrate the foundation of the optimal controls, the governing differential equations for semimajor axis a, eccentricity e, and inclination i are presented:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2 v}{\mu} a_T \cos\phi \tag{1}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{a_T}{v}\left[2(e+\cos\nu)\cos\phi + \frac{r}{a}\sin\nu\sin\phi\right] \tag{2}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{a_T r}{h}\cos\theta\sin\beta \tag{3}$$

where v is the velocity magnitude, a_T is the thrust acceleration magnitude (thrust/mass), μ is the gravitational constant, h is the angular momentum, r is the radial position magnitude, ν is the true anomaly, ω is the argument of periapsis, and $\theta = \omega + \nu$. The in-plane thrust-steering angle ϕ is measured from the velocity vector to the projection of the thrust vector onto the orbit plane, and the out-of-plane (yaw) steering angle β is measured from the orbit plane to the thrust vector. The optimal controls $\phi_a^*(t)$, $\phi_e^*(t)$, and $\beta^*(t)$ are derived from the first-order necessary (stationarity) condition for optimality for each respective variational equation [e.g., $\phi_a^*(t)$ satisfies $\partial\dot{a}/\partial\phi = 0$, etc.]. The optimal controls that maximize $\mathrm{d}a/\mathrm{d}t$, $\mathrm{d}e/\mathrm{d}t$, and $\mathrm{d}i/\mathrm{d}t$ are

$$\phi_a^* = 0 \tag{4}$$

$$\tan\phi_e^* = \frac{r\sin\nu}{2a(e+\cos\nu)} \tag{5}$$

$$\beta^* = (\pi/2)\mathrm{sgn}(\cos\theta) \tag{6}$$

The preceding controls maximize the respective variational equations because the second-order sufficient condition check yields negative values (e.g., $\partial^2\dot{a}/\partial\phi^2 < 0$). Although Eq. (6) is the optimal steering law for maximizing $\mathrm{d}i/\mathrm{d}t$, any out-of-plane steering is essentially wasted when the longitude angle θ is near ± 90 deg. Therefore, the yaw steering law

$$\beta^* = (\pi/2)\cos\theta \tag{7}$$

provides a good feedback steering law for near-maximum $+\mathrm{d}i/\mathrm{d}t$ and does not waste the thrust force near $\theta = \pm 90$ deg.

Blending the Control Laws

The in-plane thrust steering is obtained by blending the in-plane optimal controls (4) and (5). The basic steps are as follows:

1) Compute the unit vectors $\boldsymbol{c}_a$ and $\boldsymbol{c}_e$ that define the in-plane optimal controls for maximum $\mathrm{d}a/\mathrm{d}t$ and $\mathrm{d}e/\mathrm{d}t$, respectively. The unit vectors are expressed in a local rotating radial-transverse-normal (RTN) coordinate frame where the R axis is along the radial direction, the T axis is in the orbit plane along the transverse direction, and the N axis is normal to the orbit plane (however, the in-plane steering unit vectors $\boldsymbol{c}_a$ and $\boldsymbol{c}_e$ will only have R and T components). A general expression for the unit vector in the RTN frame is

$$\boldsymbol{c}_k = \left[\sin\left(\gamma + \phi_k^*\right), \cos\left(\gamma + \phi_k^*\right), 0\right]^T, \qquad k = a, e \tag{8}$$

where ϕ_a^* and ϕ_e^* are computed using the optimal controls (4) and (5). The angle γ is the flight-path angle and is measured from the local horizon to the velocity vector.

2) Combine the two in-plane thrust direction unit vectors by using the weighting functions G_a and G_e and then resolve the in-plane pitch steering angle $\alpha = \gamma + \phi$ from the blended unit vector $\boldsymbol{c}$:

$$\boldsymbol{c} = \frac{G_a \boldsymbol{c}_a + G_e \boldsymbol{c}_e}{\| G_a \boldsymbol{c}_a + G_e \boldsymbol{c}_e \|} = [\sin\alpha, \cos\alpha, 0]^T \tag{9}$$

Next, the yaw steering angle β is computed by weighting Eq. (7):

$$\beta = G_i \cos\theta \tag{10}$$

where G_i is a weighting function that scales the amplitude of the yaw steering profile.

Finally, the unit vector $\tilde{\boldsymbol{u}}$ along the thrust direction in the *RTN* frame is computed from the pitch and yaw steering angles:

$$\tilde{\boldsymbol{u}} = [\sin\alpha\cos\beta, \cos\alpha\cos\beta, \sin\beta]^T \tag{11}$$

Therefore, the thrust direction history $\tilde{\boldsymbol{u}}(t)$ is determined by weighting the control laws (4), (5), and (7). Because simplicity is often a desired attribute of a good guidance scheme, the guidance weighting functions $G_k(t)$ are parameterized by a quadratic function of time:

$$G_k(t) = K_{k0} + K_{k1}t + K_{k2}t^2, \qquad k = e, i \tag{12}$$

Therefore, the weighting functions (and subsequent thrust-direction histories) are completely determined by the guidance parameters K. Because only the relative weights of G_k are significant, the weighting function for maximum $\dot{a}$ $[G_a(t)]$ is arbitrarily set to unity for all time.

Low-Thrust Orbit Transfer

Numerical Simulation

The dynamical equations of motion for a spacecraft subject to inverse-square gravity and propulsive forces are

$$\ddot{\boldsymbol{r}} = -(\mu \boldsymbol{r}/r^3) + a_T \boldsymbol{u} \tag{13}$$

The radius vector $\boldsymbol{r} = [x, y, z]^T$ is the position of the spacecraft relative to an inertial geocentric-equatorial coordinate frame. The fundamental $(x–y)$ plane is the Earth's equatorial plane, and the $+x$ axis points in the vernal equinox direction. The thrust-direction unit vector $\boldsymbol{u}$ in the geocentric frame is determined by multiplying the thrust-direction unit vector $\tilde{\boldsymbol{u}}$ (expressed in the *RTN* frame) by a rotation matrix.

An additional differential equation for the spacecraft mass m is required:

$$\dot{m} = -\frac{2\eta P}{(g I_{sp})^2} \tag{14}$$

where η is the efficiency of the low-thrust engine, P is the input power to the engine, g is the Earth's gravitational acceleration, and I_{sp} is the specific impulse. Because an SEP spacecraft is assumed, the Earth-shadow conditions (when $P = 0$) must be determined. The details of the shadow computation are presented in Ref. 9.

The dynamical equations (13) and (14) are numerically integrated by using a standard fixed-step, fourth-order Runge–Kutta routine. The integration time step was set so that each orbital revolution resulted in at least 100 integration steps; this was found to be sufficient for acceptable numerical accuracy.

LEO–GEO Transfer

A three-dimensional LEO–GEO transfer is computed for an SEP spacecraft with the following characteristics: P is constant at 10 kW, I_{sp} is 3300 s, and η is 65%. The initial orbit (LEO) is circular with an altitude of 550 km, an inclination of 28.5 deg, and an initial ascending node angle of zero. The initial spacecraft mass is 1200 kg, and the resulting initial thrust-to-weight (T/W) ratio is $3.4(10^{-5})$. A standard GEO is the desired target orbit with $a = 42{,}164$ km, $e = 0$, and $i = 0$ deg.

Because the initial T/W ratio is so low, the orbit transfer will consist of a slowly unwinding spiral trajectory. Furthermore, as the transfer cycles between thrust and coast arcs (due to Earth shadowing), the orbit will become more eccentric. This eccentricity buildup must ultimately be removed during the final stages of the transfer. Therefore, the guidance parameters K are determined for two separate phases. During the first phase, the thrust is nearly aligned with the velocity vector (tangent steering, $\phi = 0$) with very little eccentricity control and a small out-of-plane thrust component. This simple strategy enables near-maximum gain in orbital energy so that gravity losses can be reduced. The second phase involves more emphasis on eccentricity control (to circularize the orbit) and inclination reduction.

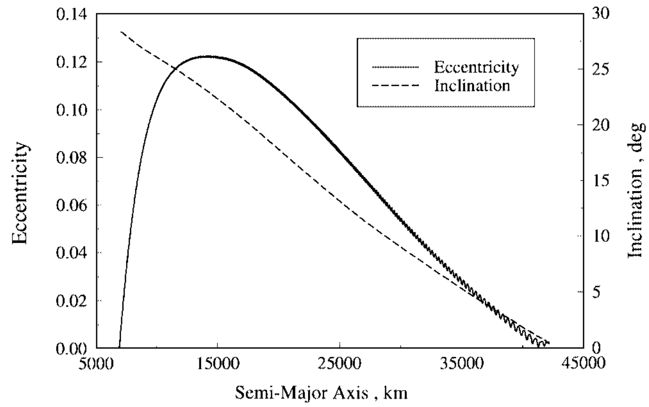

Fig. 1 Eccentricity and inclination for LEO–GEO transfer.

A successful LEO–GEO transfer was accomplished in a trip time of 216.3 days. The histories for eccentricity and inclination vs semimajor axis are presented in Fig. 1. The final values of these three orbital elements are $a = 42{,}162.3$ km, $e = 0.002$, and $i = 0.4$ deg. Two corresponding minimum-time LEO–GEO transfers were obtained in Ref. 10 for the same mission scenario by using the trajectory optimization code SEPSPOT[9] and a new direct method (DM).[10] The respective minimum trip times are 198.8 days for SEPSPOT and 200.3 days for DM. It should be noted that both SEPSPOT and DM utilize orbital averaging for the differential equations of motion (with integration step size on the order of two days), whereas the transfer presented here uses full numerical integration of the unaveraged dynamical equations with over 250,000 integration steps. Furthermore, the goal of the methods presented in this Note is not to perform the orbit transfer in minimum time but rather to develop and to demonstrate a simple guidance scheme for performing the LEO–GEO transfer in a near-optimal fashion.

Initial estimates of the guidance parameters were obtained by applying the fundamentals of three-dimensional, low-thrust orbit transfers. For example, tangent steering $(\phi = 0)$ was employed as an initial guess for the first phase of the orbit transfer by setting the maximum $\mathrm{d}e/\mathrm{d}t$ guidance parameters K_{e0}, K_{e1}, and K_{e2} to zero. Small values were assigned to the out-of-plane guidance parameters K_{i0} and K_{i1} for the first phase, and the magnitudes of these parameters were increased for the second phase so that the majority of the inclination reduction is performed as the orbital speed decreases. The GEO boundary conditions were satisfied by running the simulation and adjusting the guidance parameters so that the appropriate eccentricity and/or inclination reduction was achieved at geosynchronous altitude.

Table 1 presents the guidance parameters K for the LEO–GEO transfer. During the first phase, $0 \le t \le 120$ days, the eccentricity weighting function $G_e(t)$ decreases in a linear fashion. For the first 120 days of the transfer, the in-plane pitch steering is essentially along the velocity vector with a small component for eccentricity reduction at the end of the first phase. A quadratic fit of the eccentricity weighting function is used for the second phase $(120 \le t \le 216.3$ days) to enhance the flexibility of the guidance design. During this phase the optimal control (5) is increasingly emphasized to reduce the sizable eccentricity (see Fig. 1). The yaw steering magnitude $G_i(t)$ increases linearly throughout the transfer with two distinct linear rates. The yaw amplitude in LEO is -18.9 deg, and the yaw amplitude at GEO is -77.5 deg.

Table 1 Guidance parameters for LEO–GEO transfer

Time, days	K_{e0}	K_{e1}, day^{-1}	K_{e2}, day^{-2}	K_{i0}, rad	K_{i1}, rad/day
$0 \le t \le 120$	0.0	$-1.5(10^{-3})$	0.0	-0.33	$-4.3(10^{-3})$
$120 \le t \le 216.3$	-0.18	$-1.0(10^{-2})$	$7(10^{-5})$	-0.85	$-5.2(10^{-3})$

Note that the guidance design (the selection of the guidance parameters K) could be formalized with an iterative Newton method that results in a converged trajectory to the exact boundary conditions. However, it was found that basic engineering judgment (choosing near-tangent steering during the initial transfer, performing the plane change and eccentricity reduction at the end of the transfer, etc.) was found to be sufficient for designing a successful LEO–GEO transfer. For a real onboard guidance system, the guidance parameters K could be stored and the spacecraft could fly open-loop guidance for a majority of the transfer. A new set of guidance parameters could be recomputed based on the spacecraft's current state and uplinked at an infrequent rate (e.g., every 2–3 months). The new guidance design would account for navigation errors, dispersions, and fluctuations in engine performance. The infrequent guidance updates would allow the spacecraft to be nearly autonomous.

Conclusions

A simple guidance scheme for performing low-thrust orbit transfers is presented. The guidance is based on the optimal control laws that maximize the time rate of a respective orbital element. The guidance scheme is parameterized by scheduling weighting functions that combine the optimal controls. An LEO–GEO transfer is presented, and the proposed guidance method successfully completes the transfer with near-optimal performance. Because this guidance formulation is simple and only requires selecting a few guidance parameters (six parameters for each of the two mission phases of the LEO–GEO transfer presented here), it is well suited for an onboard implementation.

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